

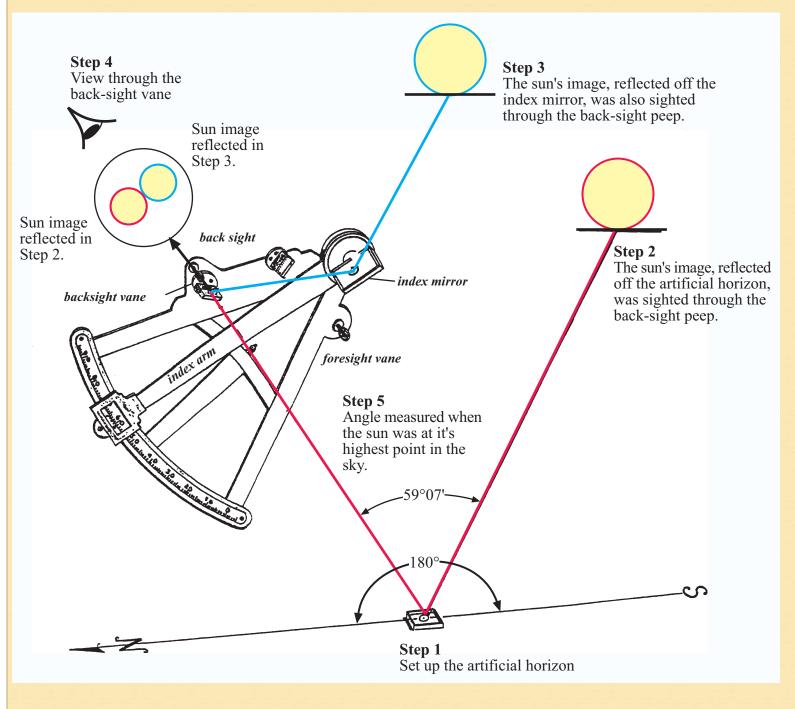
Courtesy of Independence National Historical Park

Until the latter part of the 20th Century, the most common observational method to find one's latitude was to use a sextant or $59^{\circ}07'$ octant and take a noon shot of the sun. The noon shot was preferred because 1) an accurate timepiece was not necessary and 2) an observation taken when the sun is due north or south (that is, on the observer's meridian) greatly simplifies the calculations (see Part II, below).

Since Lewis already has divided the observed angle by 2 in Step A, he must subtract that result from 90°, not 180°. Let's use one of Lewis's observations at the Three Forks to show how the captains made their observations and calculations to determine latitude.

Part I, Using the Octant to take the observation

When the sun's noon altitude was greater than 60°, Lewis and Clark used their octant, which was equipped for the back-sight method (see Equipment poster). Using the back-sight method, their octant theoretically could measure an angle between two objects up to 180° apart.



Taking a noon observation with octant and artificial horizon by the "back" method:

1) Shortly before noon on July 29, 1805, Lewis prepares to take an observation of the sun for latitude. Because mountains to the south rise above the natural horizon, Lewis uses a tray filled with water to make an artificial horizon. The water will form a level surface and reflect the sun's image to his eye.

2) Lewis places his eye to the octant's back-sight vane. He then looks through a slit in the back-sight horizon glass and finds the image of the sun reflected from the water in the tray.

3) While still sighting the sun's image reflected from the water, Lewis moves the octant's index arm with its index mirror until the sun's image is reflected from the index mirror to the backsight horizon mirror and then to his eye. Because Lewis wants to obtain the altitude of the sun's lower limb, he

brings the two images together so that the bottom of the sun's image reflected from the index mirror just touches the top of the sun's image reflected from the water.

4) As the sun continues to move toward its highest point in the sky for the day (noon), the two images of the sun overlap. To keep the two images in contact at a single point, Lewis turns the tangent screw on the octant's index arm, gradually decreasing the angle measured by the index arm.

5) Lewis keeps the two images just touching each other. When the two images stop overlapping, Lewis knows that the sun has reached its highest point in the sky. The images remain just touching each other for ten seconds or so, then begin to separate as the sun slowly descends. Lewis, however, already has clamped the index arm in place and, with the help of the vernier, reads the angle on the graduated arc as indicated by the zero mark on the index arm. That angle is 59°07'.

Part II, Applying the Corrections to the Angle Lewis Measured

The angle that Lewis measured during this noon shot is not the altitude of the sun's center but: 180° minus 2x the altitude of the sun's lower limb. The angle he measured also includes angles caused by mirror misalignment (index error), refraction (see Step D), parallax (see Step E), sun's semidiameter (the angle between the sun's lower limb and its center—see Step F) and the sun's declination (angular distance north or south of the equator—see Step G). All these angles have to be eliminated in order to derive the true altitude of the sun's center with respect to the earth's center.

The first step in making corrections to the observed angle usually is to subtract the index error (or in some manuals, apply the index correction) but . . .

A) Lewis first divides the angle he measured by 2.

Because Lewis had to measure the angle between the ray reflected from the index mirror and that from the artificial horizon, the observed angle is twice that from a natural horizon alone.

Servis and Clark in Montana Finding the Latitude Example: Three Forks of the Missouri, July 29, 1805

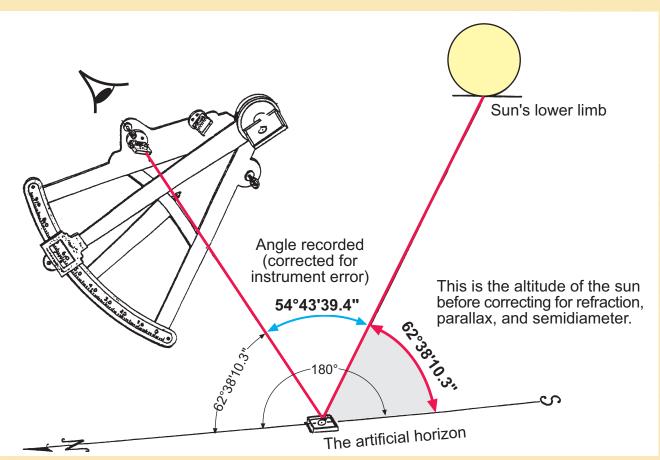
Bob Bergantino and Ginette Abdo

÷ 2 29°33'30"

B) Then he subtracts the result of Step A from 90°

90°00'00" -29°33'30" 60°26'30"

C) Next Lewis adds half the OCTANT'S INDEX ERROR



Lewis, on July 22, 1804, recorded that the octant's index error in the "back-method" was $+2^{\circ}11'40.3"$. The octant's index error actually was $+4^{\circ}23'20.6''$; that is, it read too high by that amount. Lewis, however, began his corrections by dividing his measured angle by 2 and, thus, needed to apply only half the index error (though still calling it "index error"). Because Lewis's octant read high the index error should be subtracted, but owing to the sequence Lewis used to make his corrections, he needs to add $2^{\circ}11'40.3''$ to the result of Step B.

60°26'30" +2°11'40.3" $\overline{62^{\circ}38'10.3''} = 62^{\circ}38'10'' =$ apparent altitude of sun's lower limb

Note: Although the octant's index error by Lewis's method was $+2^{\circ}11'40.3''$, he mistakenly used an index error of $+2^{\circ}40''$ throughout 1805. He discovered this mistake while at Fort Clatsop on the Pacific Coast. Using the wrong index error made the latitude he calculated on July 29 too far south by about 0°28'20" or about 32¹/₂ miles. The calculations here are made as if he had used the correct index error.

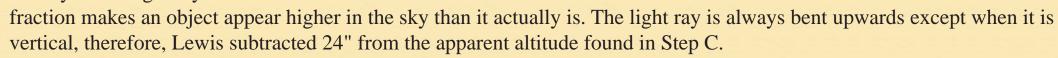
D) Then he makes the correction for REFRACTION. Lewis opens his book Tables Requisite to the table "Mean Refraction" and, using the altitude derived from Step C, he finds that he has to subtract 24".

62°38'10" apparent altitude of sun's lower limb

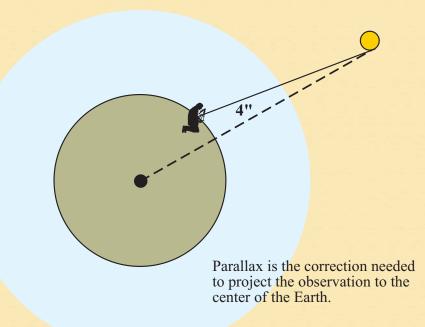
- 0°00'24" refraction correction

62°37'46" apparent altitude of sun's lower limb corrected for refraction

Refraction is the bending of a light ray as it passes through the atmosphere. This bending results from the increase in the atmosphere's density as the light ray travels downward toward the observer. Re-



E) Lewis then makes the correction for PARALLAX. The latitude that Lewis is calculating must be measured from the center of the earth. Lewis again goes to his Tables Requisite, opens it to the table "Parallax of the Sun in Altitude", and, using the apparent altitude of the sun's lower limb from Step C, finds that he has to add 4".



62°37'46" altitude of lower limb corrected for refraction +0°00'04" parallax correction 62°37'50" sun's lower limb projected to the center of the earth

F) . . . adds the SUN'S SEMIDIAMETER. It is usually easier for an observer to find either the upper or lower limb of the sun rather than its center. Lewis, when using the octant, always shot the sun's lower limb. The angular difference between the sun's center and its upper or lower limb is called its "semidiameter" (half diameter). The Nautical Almanac Lewis used had a table giving the sun's semidiameter at Greenwich noon for the 1st, 7th, 13th, 19th and 25th of each month. For July 25, 1805 it was 15'46.7" and on August 1 it was 15'47.5" For his observation on July 29, Lewis determines a semidiameter of 15'47".

The angle that Lewis has just measured includes the sun's North declination. Therefore, he must subtract the sun's declination from the result of Step F. If the declination had been south he would add it.

The Nautical Almanac for 1805 that Lewis carried, however, gave the sun's declination only for noon of each day at Greenwich, England. Lewis, thus, has to determine what the sun's changing declination would have been at the time of his observation. To do this he has to make a separate set of calculations.

How did Lewis determine the sun's declination at the time of his observation?

d) Now Lewis needs to know how many hours had elapsed between noon at Greenwich and noon at the Three Forks. The number of hours elapsed multiplied with the hourly rate gives the total change in declination since Greenwich noon. Lewis finds the number of hours elapsed by dividing his longitude by 15. He doesn't know his exact longitude, but he can estimate it or derive it by dead-reckoning. Lewis didn't save his calculations, so let's assume he used 111° West as his longitude: $111^{\circ} \div 15 = 7$ hours 24 minutes = 7.4 hours 7.4 hours x 35.7 arc seconds per hour = $0^{\circ}04'24''$

Therefore since Greenwich noon July 29 the sun's declination has decreased by 0°04'24".

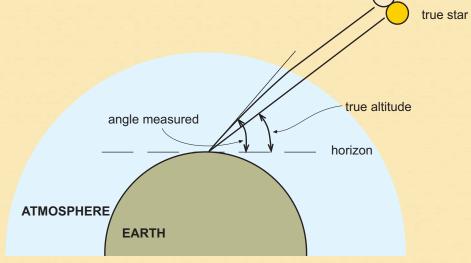
refracted star

-18°45'37" declination 44°08'00" co-latitude

H) From co-latitude to latitude. Steps A through G produce what is called a "co-latitude". A co-latitude is the angle which, when added to the latitude, equals 90° ; that is 90° - co-latitude = latitude.

To derive the calculated latitude from this observation Lewis subtracts the co-latitude from 90°

90°00'00" -44°08'00" 45°52'



Montana Bureau of Mines and Geology A Department of Montana Tech of the University of Montana





William Clark

Courtesy of Independence National Historical Park

sun's center sun's lower limb

He adds this to the result of Step E to find the true altitude of its center because he shot the sun's lower limb.

62°37'50" altitude of lower limb projected to the center of the earth $+0^{\circ}15'47''$ sun's semidiameter

62°53'37" true altitude of the sun with respect to the earth's center

G) . . . and subtracts the SUN'S DECLINATION. Now Lewis has the true altitude of the sun's center above the horizon, but that altitude includes the sun's north declination.

A celestial body's declination is the number of degrees that body is north or south of the celestial equator at a given moment as "viewed"

from the center of the earth. The declination of stars and distant planets changes slowly, but the sun's declination is constantly changing except for a brief instant at the summer and winter solstice. From the spring equinox until the autumnal equinox the sun's declination is North (+); the remainder of the year its declination is South (-).

a) Lewis looks in his Nautical Almanac for July. On page 74 in the column with the heading "Day of the Month" he finds "29" and following across to the right to the column with the heading "Declin. North"

and for the 30th he finds18°35'44" (July 30) 0°14'17"change per day b) He subtracts the declination for July 30 from that for July 29 c) Then he divides that result by 24 to find the change in ÷24 0°00'35.7" change per hour sun's declination per hour

- e) Lewis subtracts this amount from the sun's noon declination
- +18°50'01" declination at Greenwich noon
- 0°04'24" total change
- +18°45'37" declination Three Forks at local noon

Finally, he subtracts the sun's declination from the result of Step F

- 62°53'37" from Step F

45°52'00" LATITUDE

Latitude to nearest arc minute per this observation (about 3 miles too far south)

45°55¹/2' Latitude of camp determined from map analysis

45°23'23" Lewis's calculated latitude using incorrect index error of 2°40' instead of 2°11'40"

